A Time Variation of Proton-Electron Mass Ratio and Grand Unification

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May, 2006

Abstract

Astrophysical observations indicate a time variation of the proton-electron mass ratio and of the fine-structure constant. We discuss this phenomenon in models of Grand Unification. In these models a time variation of the fine-structure constant and of the proton mass are expected, if either the unified coupling constant or the scale of unification changes, or both change. We discuss in particular the change of the proton mass. Experiments in Quantum Optics could be done to check these ideas.

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A new indication that fundamental constants of nature could evolve with the cosmological time has recently been published [1]. The ratio of the proton mass to electron mass $\mu = m_p/m_e$ was bigger in the past and has decreased over the last 12 Gyr. The authors of [1] report

$$\frac{\Delta\mu}{\mu} = (2.4 \pm 0.6) \times 10^{-5} \tag{1}$$

which after the result of Webb *et al.*

$$\frac{\Delta\alpha}{\alpha} = (-0.72 \pm 0.18) \times 10^{-5} \tag{2}$$

for a redshift $z \approx 0.5 \ldots 3.5$ [2], which indicates a possible time dependence of the fine structure constant α, is the second indication of a possible time dependence of fundamental parameters of nature. We note however that the result of Webb *et al.* could not be reproduced by similar observations [3–7].

In [8–10] (see also [11–16] for related works) we used (2) to predict the parameter $\frac{\Delta\mu}{\mu}$ within the framework of a simple unified theory and found in the simplest case that a time variation of the unification scale could give rise to an effect of the order of 10^{-4}. In this letter we want to point out that a variation of the unification scale combined with that of the unification coupling constant and/or of the supersymmetry breaking scale could easily lead to an effect of the order of 10^{-5} as observed in [1] .

As in [8–10], we consider as an example the supersymmetric extension of the standard model. This model can be unified in the framework of SO(10). Supersymmetry is not required to have the unification of the gauge couplings (see e.g. [17,18]), but it offers a simple model for our discussion. We work in the one loop approximation. Time variations of Yukawa and Higgs boson masses can be neglected within this approximation. Furthermore not all time variations of physical scales can be observables, only ratios of scales or dimensionless quantities can be observed. Within these approximations, we only have three parameters: the unification scale Λ_u, the unified coupling constant α_u and the scale of the supersymmetry breaking Λ_S. We note that the proton mass is determined mainly by the QCD scale. Quark masses do not play a big role. Furthermore, QED splits the neutron and proton masses, besides the quark masses, and this ratio could be time dependent and lead to an observable effect .

We now focus on the QCD scale Λ_{QCD} and extract its value from the Landau pole of the renormalization group equations for the couplings of the supersymmetric standard model

$$\begin{aligned}
\alpha_3(\mu)^{-1} \quad &= \quad \left(\frac{1}{\alpha_u(\Lambda_u)} + \frac{1}{2\pi} b_3^S \ln\left(\frac{\Lambda_u}{\mu}\right)\right)\theta(\mu - \Lambda_S) \\
&\quad + \left(\frac{1}{\alpha_3(\Lambda_S)} + \frac{1}{2\pi} b_3 \ln\left(\frac{\Lambda_S}{\mu}\right)\right)\theta(\Lambda_S - \mu),
\end{aligned} \tag{3}$$

where the parameters b_i are given by $b_i = (b_1, b_2, b_3) = (41/10, -19/6, -7)$ and by $b_i^S = (b_1^S, b_2^S, b_3^S) = (33/5, 1, -3)$ and where $\alpha_i(\Lambda_S)$ is the value of the coupling constant at the supersymmetry breaking scale which can be expressed in terms of Λ_S, α_u and Λ_u [1]. The QCD scale is given by

$$\Lambda_{QCD} = \Lambda_S \left(\frac{\Lambda_u}{\Lambda_S}\right)^{\frac{b_3^S}{b_3}} \exp\left(\frac{2\pi}{\alpha_u}\right)^{\frac{1}{b_3}}. \tag{4}$$

The time variation of Λ_{QCD} is then determined by

$$\frac{\dot{\Lambda}_{QCD}}{\Lambda_{QCD}} = -\frac{2\pi}{b_3}\frac{\dot{\alpha}_u}{\alpha_u^2} + \frac{b_3^S}{b_3}\frac{\dot{\Lambda}_u}{\Lambda_u} + \frac{b_3 - b_3^S}{b_3}\frac{\dot{\Lambda}_S}{\Lambda_S}. \tag{5}$$

This equation determines the ratio $\frac{\Delta\mu}{\mu}$, since we keep the electron mass constant. We thus find

$$\frac{\Delta\mu}{\mu} = -\frac{2\pi}{b_3}\frac{\dot{\alpha}_u}{\alpha_u^2} + \frac{b_3^S}{b_3}\frac{\dot{\Lambda}_u}{\Lambda_u} + \frac{b_3 - b_3^S}{b_3}\frac{\dot{\Lambda}_S}{\Lambda_S} = \frac{2\pi}{7}\frac{\dot{\alpha}_u}{\alpha_u^2} + \frac{3}{7}\frac{\dot{\Lambda}_u}{\Lambda_u} - \frac{4}{7}\frac{\dot{\Lambda}_S}{\Lambda_S}. \tag{6}$$

This equation is rather instructive. A cancellation between the functions $\frac{\dot{\alpha}_u}{\alpha_u^2}$, $\frac{\dot{\Lambda}_u}{\Lambda_u}$ and $\frac{\dot{\Lambda}_S}{\Lambda_S}$ could easily occur and give $\frac{\Delta\mu}{\mu} \sim 2 \times 10^{-5}$. If we assume the result eq. (2) for the fine-structure constant and take only a time dependence of the unification scale or of the unified coupling constant in account, we find [9] for the mass ratio either $\Delta\mu/\mu \sim 22 \times 10^{-5}$ (for $\alpha_u = \text{const.}$) or $\Delta\mu/\mu \sim -27 \times 10^{-5}$ (for $\Lambda_u = \text{const.}$). Thus one can obtain the observed result (1) by having a cancellation in eq. (6).

It is interesting to point out that the measurement (2) provides a direct determination of the time dependence of the unified coupling constant:

$$\frac{\dot{\alpha}_u}{\alpha_u^2} = \frac{3}{8}\frac{\dot{\alpha}}{\alpha^2} \tag{7}$$

since this relation is renormalization scale invariant. We find $\dot{\alpha}_u/\alpha_u^2 = -3 \times 10^{-14}\text{yr}^{-1}$ using (2) as an input, i.e. the unified coupling constant was smaller in the past. If we assume that the supersymmetry scale is time independent, we find that the new measurement (1) together with (2) allows to extract the time variation of the unification scale:

$$\frac{\dot{\Lambda}_u}{\Lambda_u} = \left(\frac{\dot{\Lambda}_{QCD}}{\Lambda_{QCD}} + \frac{2\pi}{b_3}\frac{3}{8}\frac{\dot{\alpha}}{\alpha^2}\right)\frac{b_3}{b_3^S}. \tag{8}$$

[1] We would like to point our that our formalism is not dependent on whether nature is supersymmetric or not at some higher scale. Indeed the same formalism would apply to an SO(10) gauge symmetry broken to the standard model with an intermediate gauge symmetry. In that case the b_i^S would be the beta-functions of the intermediate gauge symmetry and Λ_S the energy scale of the intermediate gauge symmetry breaking.

We find $\dot{\Lambda}_u/\Lambda_u = 7 \times 10^{-14}\mathrm{yr}^{-1}$, i.e. the unification scale was higher in the past.

The new observation (1) implies a time variation for the proton-electron mass ratio $\frac{\Delta\mu}{\mu}$ of the order of $2 \times 10^{-15}\mathrm{yr}^{-1}$, if linearly extrapolated, which should be observable in quantum optics experiments using modern techniques. This is in contrast to the expectation of [9] where a time variation of the grand unification scale only would imply a change of $\frac{\Delta\mu}{\mu} \sim 3 \times 10^{-14}\mathrm{yr}^{-1}$ which is now experimentally excluded.

A time variation of 2×10^{-15} per year can be observed by precise experiments in quantum optics, e.g. by comparing a cesium clock with hydrogen transitions, as done in ref. [19]. In a cesium clock the time is measured by using a hyperfine transition. The frequency of the clock depends therefore on the magnetic moment of the cesium nucleus. The latter is directly proportional to Λ_{QCD}. The hydrogen transitions, however, are only dependent on the electron mass, which we assume to be constant.

A time variation of the QCD scale and of the unification scale of the order discussed above would imply that considerable changes of physics are expected at times very close to the Big Bang. For example, the results for nucleosynthesis will be changed (see e.g. [20,21]). However the details are highly model dependent and beyond the scope of this paper.

Acknowledgments

This work was supported in part by the IISN and the Belgian science policy office (IAP V/27).

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